Spinor algebra transformations as gauge symmetry: limit to Einstein gravity

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Abstract

We construct a Lagrangian of Weyl spinors and gauge fields, which is invariant under the action of equivalent local transformations on the spinor algebra representations. A model of vacuum with a nontrivial gauge strength-tensor setting a scale and spontaneously breaking the gauge symmetry is suggested, so that the leading approximation at low energies is described by the Einstein–Hilbert Lagrangian of gravity. We consider a mechanism for the cancellation of cosmological constant due to a symmetry between the vacuum strength tensor and the tensor dual to it. The appearance of nonzero masses for the non-graviton degrees of freedom for the gauge field is shown. The generalization to the Dirac spinors is considered, and a group of additional gauge symmetry is determined. We argue for a necessary introduction of supersymmetry.

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1 Introduction

For two-component Weyl spinors θ_α the algebra of σ-matrices is defined by the commutation relations

$$\begin{aligned} \{\sigma^\mu\bar\sigma^\nu+\sigma^\nu\bar\sigma^\mu\}_\alpha^{\ \beta} &= 2\eta^{\mu\nu}\,\delta_\alpha^\beta, \\ \{\bar\sigma^\mu\sigma^\nu+\bar\sigma^\nu\sigma^\mu\}^{\dot\alpha}_{\ \dot\beta} &= 2\eta^{\mu\nu}\,\delta^{\dot\alpha}_{\dot\beta}, \end{aligned} \tag{1}$$

where $\eta^{\mu\nu} = \mathrm{diag}[1,-\mathbf{1}]$ is the metric tensor of Minkowski, and the Hermitian matrices $\sigma^\mu = (1,\boldsymbol{\sigma})$ have the explicit indices: $\sigma^\mu_{\alpha\dot\alpha}$ and $\bar\sigma^{\mu\dot\alpha\alpha} = \sigma^\mu_{\beta\dot\beta}\,\epsilon^{\dot\alpha\dot\beta}\,\epsilon^{\alpha\beta}$, so that ϵ is completely anti-symmetric tensor with the normalization $\epsilon^{12}=1$, and the spinor indices are defined in accordance with the following prescriptions accepted in [1]:

θ_α is the two-component left-handed spinor-column,

$\bar\theta_{\dot\alpha} = [\theta_\alpha]^\dagger$ is the Hermitian-conjugated spinor-row,

$\bar\theta^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}[\theta_\beta]^\star = \theta_c$ is the charge-conjugated spinor-column,

$\theta^\alpha = \epsilon^{\alpha\beta}\theta_\beta = [\theta_c]^\dagger$ is the Hermitian-conjugated spinor-row of charge-conjugated spinor.

Then the invertible complex matrices with the unit determinant $M \in SL(2, \boldsymbol{C})$ transform the spinors in the following way:

$\theta'_\alpha = M_\alpha^{\ \beta}\,\theta_\beta$, or in the matrix notations $\theta' = M\theta$,

$\bar\theta'_{\dot\alpha} = M^{\star\ \dot\beta}_{\ \dot\alpha}\,\bar\theta_{\dot\beta}$, i.e. $\bar\theta' = \bar\theta M^\dagger$,

$\bar\theta'^{\dot\alpha} = [M^\star]^{-1\ \dot\alpha}_{\ \dot\beta}\,\bar\theta^{\dot\beta}$, i.e. $\theta'_c = [M^\dagger]^{-1}\theta_c$,

$\theta'^\alpha = [M^{-1}]_\beta^{\ \alpha}\,\theta^\beta$, i.e. $[\theta'_c]^\dagger = [\theta_c]^\dagger M^{-1}$,

so that we can easily show that the products of $\theta_c^\dagger\theta = \theta^\alpha\theta_\alpha$ and $\theta^\dagger\theta_c = \bar\theta_{\dot\alpha}\bar\theta^{\dot\alpha}$ are invariant. The matrices M generate the equivalent transformations of σ-matrix representations, conserving the form of commutation relations (1):

$$\sigma'^\mu = M\,\sigma^\mu\,M^\dagger, \quad \bar\sigma'^\mu = [M^\dagger]^{-1}\,\bar\sigma^\mu\,M^{-1}. \tag{2}$$

The infinitesimal transformations are given by six components of anti-symmetric tensor ω_{nm}, where $\{n, m\} = (0, 1, 2, 3)$, so that

$$\begin{array}{rcl} M &=& 1 + \sigma^{nm}\omega_{nm}, \\ [M^\dagger]^{-1} &=& 1 + \bar\sigma^{nm}\omega_{nm}, \end{array} \qquad \omega_{nm} \to 0, \tag{3}$$

and the generators of transformations are defined by the following relations:

$$\begin{array}{rcl} [\sigma^{nm}]_\alpha^{\ \beta} &=& \frac{1}{4}\left[\sigma^n_{\alpha\dot\alpha}\bar\sigma^{m\dot\alpha\beta} - \sigma^m_{\alpha\dot\alpha}\bar\sigma^{n\dot\alpha\beta}\right], \\ [\bar\sigma^{nm}]^{\dot\alpha}_{\ \dot\beta} &=& \frac{1}{4}\left[\bar\sigma^{n\dot\alpha\beta}\sigma^m_{\beta\dot\beta} - \bar\sigma^{m\dot\alpha\beta}\sigma^n_{\beta\dot\beta}\right]. \end{array} \tag{4}$$

In contrast to the space-time transformations by the Lorentz group, which are given by the tensor of busts and rotations ω and act on the Weyl spinors by the corresponding matrices of $M(\omega)$ in the theory of special relativity, let us consider the equivalent transformations of spinor algebra representations as a gauge group, which action does not change the space-time coordinates. Such the gauge group changes the explicit form of σ-matrices, which can differ from the standard definition of Pauli matrices. The observables of free Weyl particle do not depend on the operation of this group, of course, i.e. the group transformations conserve the invariant action of Weyl spinor:

$$\begin{array}{rcl} S &=& \int d^4x\,\mathcal{L}_0, \\ \mathcal{L}_0 &=& \frac{i}{2}[\bar\theta(x)\,\bar\sigma^\mu\partial_\mu\theta(x) + \theta(x)\,\sigma^\mu\partial_\mu\bar\theta(x)]. \end{array} \tag{5}$$

Thus, we study the principle of relativity for the choice of basis system in the algebra of σ-matrices for the Weyl spinors.

In the present paper we consider the global transformations of algebra representations for the Weyl spinors in the Minkowskian space-time and derive the corresponding Noether currents in section 2. These currents determine the form of interaction with external sources,

so that this interaction has the form of product for the current and spin connection, which appears in the description of gravitational interaction for the particles possessing the spin in addition to the contact term of energy-momentum tensor with the metrics. Further we study the local gauge invariance in the spinor algebra and introduce the covariant derivative on the left-handed and right-handed Weyl spinors in section 3. The commutator of covariant derivatives determines the tensor of gauge field strength, which coincides with the curvature tensor of spin-connection. We formulate the action of gauge connection-field in the Minkowskian space-time in terms of standard Lagrangian written down as the square of strength tensor. This Lagrangian allows the expression in the form of trace for the square of gauge field strength tensor in the algebra of generators on the left-handed and right-handed spinors. We assume that the Lagrangian of spin-connection gauge field interacting with the Weyl spinors is renormalizable[1] in the Minkowskian space-time, as it usually takes place in the gauge theories. The invariance of Lagrangian under the action of group is provided by introducing the gauge transformation of tetrad (vierbein), which determine the relation between the basis in the spinor algebra and the system of Minkowskian space-time coordinates. As a consequence of tetrad transformation, the energy-momentum tensor by its anti-symmetric part enters the law for the conservation of spinor current. The local symmetry leads to the introduction of *auxiliary* field of tetrad[2]. However, we have two kinds of coordinate indices related to the "world" points and local Minkowskian reference-system connected to the algebra of σ-matrices. Therefore, the invariant measure of space-time volume defined under the "world" coordinates should contain the usual factor of $\det[h]$, where h denotes the tetrad, which can determine a Riemannian metric.

The idea explored in the present paper is connected to that of obtaining the general relativity by gauging the Lorentz group as date back to Utiyama [3], and that approach has been widely used in supergravity, where it is the simplest way to find the appropriate invariants. The essential difference of present consideration from that of Utiyama is that we do not deal with the transformation of coordinates, while we investigate the local symmetry, which does not act on the space-time variables. A similar model of extended Yang–Mills gauge theory in Euclidean space was considered in [4], where the author focused on the description of higher spins with renormalizable interactions including the supersymmetry and spontaneous breaking of local symmetry in the Higgs mechanism with scalar particles, which is rather different from the approach explored in the persent paper. So, the material upto section 3.1 gives a presentation of rather known things as suitable for the purposes of current study.

Further we involve an assumption on a gauge field condensation and the spontaneous breaking of gauge symmtery, that results in a background strength tensor in the classic theory. Particularly, we write down an example of global, independent of coordinates spin-connection with a covariant expectation value of vacuum field, that can be related with the expansion of strength tensor in terms of sum for the strength tensor of vacuum background field and the dynamical tensor of curvature. The contribution linear over the background strength and dynamical curvature coincides with the form of Lagrangian in the Einstein–Hilbert theory of gravitation. Thus, we see that the action of general relativity occurs as the result of the gauge

[1] The renormalizability of gravity with the terms quadratic in the curvature tensor was demonstrated by K.S.Stelle in [2].

[2] In this way, the corresponding tensor of curvature under the metric connection generally can remain equal to zero, and the space-time of Minkowski is not curved as referred to the spinor algebra.

field condensation and the spontaneous breaking of gauge symmtery, and in this limit it is the effective low-energy contribution into the full action, that leads to the curved space-time because of the vacuum fields. Note, that the condensation conserves the local gauge symmetry and, hence, the local Lorentz invariance takes place as well as the symmetry under the general coordinate transformations does, so that the invariant measure in terms of "world" indices naturally contains the ordinary factor of $\det[h]$. The term of Lagrangian quadratic over the dynamical strength tensor becomes significant at the energy of spin-connection quanta about the Planck scale in the vicinity of which, probably, the decondensation takes place, and therefore the space-time becomes flat under the absence of vacuum fields. The Lagrangian term quadratic over the background connection strength tensor generally leads to a cosmological constant, which is cancelled by the contribution caused by the product of background connection and spinor current, if we assume that there is a corresponding condensate of spinor field. The complete cancellation of these two terms can be broken by a fluctuation of fields, that naturally leads to an inflation expansion or contraction of universe depending on the sign of fluctuation. We show that in the studied example of background connection, the dynamical modes of excitations have the effective masses in the region of Planck scale. So, we assume that the modes of gauge field except the graviton aer suppressed outside the Planck scale in the analogous way to the gluons in QCD, and these modes can be confined. Further we consider some problems appearing in the formulation of Hamiltonian dynamics of spinor-connection gauge field and suggest an ansatz in terms of dual fields, that allows us to classify the dynamical modes of field and to offer an alternative mechanism for the cancellation of cosmological constant due to a symmetry between the vacuum gauge field and the field in the dual strength tensor. We comment also the problem on how the vacuum fields enter the field equations.

In section 4 we briefly discuss modifications of approach under consideration in the case of Dirac field, for which we naturally introduce an additional gauge symmetry including the electroweak symmetry of standard model. In section 5 we discuss a reason for the correlation of vacuum bosonic and fermionic fields in the light of supersymmetry, that, probably, provides the cancellation of cosmological constant. We emphasize that the introduction of supersymmetry is a necessary consequence of BRST-generalization of evolution operator for the spinor field. Finally, we discuss the obtained results of suggested approach in conclusion.

2 Global symmetry. Noether currents

Following the standard procedure, let us determine the Noether currents according to the formula

$$j_a^\mu = \frac{\delta\mathcal{L}}{\delta\partial_\mu\psi}\,\frac{\partial\psi}{\partial\omega^a} + \frac{\delta\mathcal{L}}{\delta\partial_\mu\bar{\psi}}\,\frac{\partial\bar{\psi}}{\partial\omega^a}, \tag{6}$$

where in the case under study we put $\omega^a \to \omega_{nm}$, the parameters of infinitesimal transformations (3) for the spinors θ and $\bar{\theta}$. Then in the explicit form we find

$$j^{\mu,nm} = \frac{i}{2}[\bar{\theta}(x)\,\bar{\sigma}^\mu\,\sigma^{nm}\theta(x) + \theta(x)\,\sigma^\mu\,\bar{\sigma}^{nm}\bar{\theta}(x)]. \tag{7}$$

Using the Pauli gauge for the σ-matrices, $\sigma^\mu = (1, \boldsymbol{\sigma})$, we can show that

$$j^{\mu,nm} = \frac{1}{2}\,\epsilon^{\mu nm\nu}\,j^\lambda\,\eta_{\nu\lambda}, \tag{8}$$

where $j^\lambda = \bar\theta\,\bar\sigma^\lambda\,\theta$ is the spinor current, which is rotated under the action of group transformations in the space of σ-matrices.

Under the action of global transformation we get the spinor Lagrangian equal to

$$\begin{aligned}\mathcal{L}'_0 &= \tfrac{i}{2}[(\bar\theta M^\dagger)\,([M^\dagger]^{-1}\bar\sigma^\mu M^{-1})\,\partial_\mu(M\theta) + (\theta M^{-1})\,(M\sigma^\mu M^\dagger)\,\partial_\mu([M^\dagger]^{-1}\bar\theta)] \\ &= \tfrac{i}{2}[\bar\theta'\,\bar\sigma'^\mu\partial_\mu\theta' + \theta'\,\sigma'^\mu\partial_\mu\bar\theta'] = \mathcal{L}_0,\end{aligned} \tag{9}$$

and it is quite evident that the variation of Lagrangian is equal to zero, $\mathcal{L}'_0 - \mathcal{L}_0 = 0$. Let us introduce the tetrad $h_m^{\ \mu}$, which relates the local "latin" indices in the basis of σ-matrices with the world "greek" indices of space-time:

$$\begin{array}{l}\sigma^\mu = \sigma^m\,h_m^{\ \mu}, \quad \bar\sigma^\mu = \bar\sigma^m\,h_m^{\ \mu}, \quad h_m^{\ \mu}\,h_n^{\ \nu}\,\eta^{mn} = \eta^{\mu\nu}, \\ \sigma^m = \sigma^\mu\,h_\mu^{\ m}, \quad \bar\sigma^m = \bar\sigma^\mu\,h_\mu^{\ m}, \quad h^\mu_{\ m}\,h^\nu_{\ n}\,\eta_{\mu\nu} = \eta_{mn},\end{array} \qquad h_m^{\ \mu}\,h^\nu_{\ n}\,\eta_{\mu\nu} = \eta_{mn}, \tag{10}$$

so that for the transformed σ-matrices we get

$$\sigma'^\mu = \sigma^m\,h'^{\ \mu}_m, \tag{11}$$

and the infinitesimal rotation of tetrad has the form

$$\delta h_m^{\ \mu} = h_n^{\ \mu}\,\omega_{lm}\,\eta^{nl}. \tag{12}$$

The variation of inverse tetrad $h^\nu_{\ n}$ can be written down in accordance with (10)

$$\delta h^\nu_{\ n} = -h^{\nu m} h_{\mu n}\,\delta h_m^{\ \mu}, \tag{13}$$

where the tetrad indices are moved down and up by the metric tensor and that of inverse to it, correspondingly.

Then the variation of Lagrangian can be written down in the form

$$\delta\mathcal{L} = \delta\omega_{nm}\,[\partial_\mu\,j^{\mu,nm} + h^{n\mu}T_\mu^{\ m}], \tag{14}$$

so that the invariance of Lagrangian under the global transformations leads to the conservation law

$$\partial_\mu\,j^{\mu,mn} + \frac{1}{2}(T^{mn} - T^{nm}) = 0. \tag{15}$$

As we see, in the absence of sources the current is conserved with the accuracy up to the anti-symmetric part of energy-momentum tensor T^{nm} for the spinor field. This tensor could be generally reduced to the symmetric one due to the additional term of Lagrangian in the form of divergence for a current, so that this term is transformed to the surface integral and the equations of motion remain with no changes.

After the introduction of sources the interaction Lagrangian has the form[3]

$$\mathcal{L}_{\rm int} = j^{\mu,nm}\,\mathcal{A}_{\mu,nm}. \tag{16}$$

In this way the source, i.e. the spin-connection, should possess some transformation properties under the action of symmetry group for the full action remains invariant after the change of spinor algebra representation. These properties of source are investigated in the study of local symmetry.

[3]We see that the tetrad is not a source for a Noether current, and therefore, it is an auxiliary field.

3 Local symmetry

The generators of spinor transformations satisfy the standard commutation relations

$$\begin{aligned}
[\sigma^{nm},\, \sigma^{kl}] &= -\eta^{nk}\sigma^{ml} + \eta^{nl}\sigma^{mk} - \eta^{ml}\sigma^{nk} + \eta^{mk}\sigma^{nl}, \\
[\bar\sigma^{nm},\, \bar\sigma^{kl}] &= -\eta^{nk}\bar\sigma^{ml} + \eta^{nl}\bar\sigma^{mk} - \eta^{ml}\bar\sigma^{nk} + \eta^{mk}\bar\sigma^{nl},
\end{aligned} \tag{17}$$

so that we introduce the "covariant" derivatives on the left-handed and right-handed spinor fields

$$\begin{aligned}
\nabla_{\mu\alpha}{}^{\beta} &= \delta_\alpha^{\ \beta}\,\partial_\mu + \mathcal{A}_{\mu,nm}\sigma^{nm}{}_{\alpha}{}^{\beta}, \\
\overline{\nabla}_{\mu}{}^{\dot\alpha}{}_{\dot\beta} &= \delta^{\dot\alpha}_{\ \dot\beta}\,\partial_\mu + \mathcal{A}_{\mu,nm}\bar\sigma^{nm\dot\alpha}{}_{\dot\beta},
\end{aligned} \tag{18}$$

which have the consistent commutator determining the strength tensor of gauge field $\mathcal{A}$

$$\begin{aligned}
[\nabla_\mu,\, \nabla_\nu] &= \mathcal{F}_{\mu\nu,mn}\,\sigma^{mn}, \\
[\overline{\nabla}_\mu,\, \overline{\nabla}_\nu] &= \mathcal{F}_{\mu\nu,mn}\,\bar\sigma^{mn},
\end{aligned} \tag{19}$$

where

$$\mathcal{F}_{\mu\nu,mn} = \partial_\mu \mathcal{A}_{\nu,mn} - \partial_\nu \mathcal{A}_{\mu,mn} + 2(\mathcal{A}_{\mu,mk}\mathcal{A}_{\nu,ln} - \mathcal{A}_{\nu,mk}\mathcal{A}_{\mu,ln})\,\eta^{kl}. \tag{20}$$

In the derivation of (20) we have explored the anti-symmetry of spin-connection over the group indices $\mathcal{A}_{\mu,nm} = -\mathcal{A}_{\mu,mn}$. We emphasize that the strength tensor is reduced to the standard form of curvature tensor for the connection determined by $\Gamma_{\mu,nm} = 2\mathcal{A}_{\mu,nm}$, so that $\mathcal{F}_{\mu\nu,mn}[\mathcal{A}] = \frac{1}{2}\mathcal{R}_{\mu\nu,nm}[\Gamma]$.

Define the Lagrangian of gauge field $\mathcal{A}$ in the space-time of Minkowski in the ordinary form[4]

$$\mathcal{L}_{\mathcal{A}} = \frac{1}{4g_{\rm Pl}^2}\,\mathcal{F}_{\mu\nu,mn}\mathcal{F}^{\mu\nu,mn}, \tag{21}$$

where $g_{\rm Pl}$ is a coupling constant of gauge field. Let us show that Lagrangian (21) is invariant under the action of local gauge transformations giving the equivalent representations of spinor algebra, if we introduce the following transformations of gauge fields:

$$\mathcal{A}'_{\mu,nm}\,\sigma^{nm} = M^{-1}\mathcal{A}_{\mu,nm}\,\sigma^{nm}\,M - (\partial_\mu M^{-1})\cdot M, \tag{22}$$

which follows from the definition of covariant derivative

$$\begin{aligned}
M\,\nabla_\mu(\mathcal{A}')\,M^{-1} &\stackrel{\rm def}{=} \nabla_\mu(\mathcal{A}), \\
[M^\dagger]^{-1}\,\overline{\nabla}_\mu(\mathcal{A}')\,M^\dagger &\stackrel{\rm def}{=} \overline{\nabla}_\mu(\mathcal{A}).
\end{aligned} \tag{23}$$

Definitions (23) are consistent, i.e. they result in the same transformation of field $\mathcal{A}$ (22), since the relation $[\sigma^{nm}]^\dagger = -\bar\sigma^{nm}$ is valid. Then, definition (19) leads to the group transformation of strength tensor in the algebra of left-handed and right-handed spinors in the following form:

$$\begin{aligned}
M\,\mathcal{F}_{\mu\nu,mn}(\mathcal{A}')\,\sigma^{mn}\,M^{-1} &= \mathcal{F}_{\mu\nu,mn}(\mathcal{A})\,\sigma^{mn} = \mathcal{F}_{\mu\nu}, \\
[M^\dagger]^{-1}\,\mathcal{F}_{\mu\nu,mn}(\mathcal{A}')\,\bar\sigma^{mn}\,M^\dagger &= \mathcal{F}_{\mu\nu,mn}(\mathcal{A})\,\bar\sigma^{mn} = \overline{\mathcal{F}}_{\mu\nu}.
\end{aligned} \tag{24}$$

[4] The sign in front of strength tensor squared is fixed in the consistent way along with the definition of covariant derivative.

Thus, the traces over the spinor indices for the squares of following quantities are invariant:

$$
\begin{aligned}
\mathrm{Tr}[\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}] &= -\tfrac{1}{2}[\delta_k^m\delta_l^n - \delta_l^m\delta_k^n - i\epsilon^{mn}{}_{kl}]\,\mathcal{F}_{\mu\nu,mn}\mathcal{F}^{\mu\nu,kl},\\
\mathrm{Tr}[\overline{\mathcal{F}}_{\mu\nu}\overline{\mathcal{F}}^{\mu\nu}] &= -\tfrac{1}{2}[\delta_k^m\delta_l^n - \delta_l^m\delta_k^n + i\epsilon^{mn}{}_{kl}]\,\mathcal{F}_{\mu\nu,mn}\mathcal{F}^{\mu\nu,kl}.
\end{aligned}
\tag{25}
$$

Then we can write down the Hermitian-conjugated expression

$$
\begin{aligned}
\mathcal{L}_{\mathcal{F}} &= -\mathfrak{Re}\,\frac{1}{8\mathfrak{g}_{\mathrm{Pl}}^2}\,\mathrm{Tr}[\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}] - \mathfrak{Re}\,\frac{1}{8\mathfrak{g}_{\mathrm{Pl}}^{\star 2}}\,\mathrm{Tr}[\overline{\mathcal{F}}_{\mu\nu}\overline{\mathcal{F}}^{\mu\nu}]\\
&= \frac{1}{4g_{\mathrm{Pl}}^2}\,\mathcal{F}_{\mu\nu,mn}\mathcal{F}^{\mu\nu,mn} + \frac{\theta}{32\pi^2}\epsilon^{mn}{}_{kl}\,\mathcal{F}_{\mu\nu,mn}\mathcal{F}^{\mu\nu,kl},
\end{aligned}
\tag{26}
$$

where we have introduced the notations for the real and imaginary parts of inverse charge $\mathfrak{g}_{\mathrm{Pl}}$ squared, so that

$$
\mathfrak{Re}\frac{1}{4\mathfrak{g}_{\mathrm{Pl}}^2} = \frac{1}{4g_{\mathrm{Pl}}^2}, \qquad \mathfrak{Im}\frac{1}{4\mathfrak{g}_{\mathrm{Pl}}^2} = \frac{\theta}{16\pi^2}. \tag{27}
$$

In Lagrangian (26) we will not consider the "θ-term", which is usually introduced in the case of nontrivial structure of vacuum caused by instantons[5].

Then, the Lagrangian invariant under the local transformations of spinor algebra representations has the form

$$
\mathcal{L} = \frac{1}{4g_{\mathrm{Pl}}^2}\,\mathcal{F}_{\mu\nu,mn}\mathcal{F}^{\mu\nu,mn} + \frac{i}{2}[\bar{\theta}(x)\,\bar{\sigma}^\mu\nabla_\mu\theta(x) + \theta(x)\,\sigma^\mu\overline{\nabla}_\mu\bar{\theta}(x)]. \tag{28}
$$

3.1 Identities of Slavnov–Taylor

The law of spinor current conservation in the presence of gauge field takes the form

$$
\partial_\mu\, j^{\mu,mn} + j^{\nu,kl}\,\left.\frac{\delta\mathcal{A}_{\nu,kl}}{\delta\omega_{mn}}\right|_{\mathrm{glob}} + \frac{1}{2}(T^{mn} - T^{nm}) = 0, \tag{29}
$$

so that we introduce the covariant divergence of current by the following definition:

$$
[\nabla_\mu\, j^\mu]^{nm} = \partial_\mu\, j^{\mu,nm} + j^{\nu,kl}\,\left.\frac{\delta\mathcal{A}_{\nu,kl}}{\delta\omega_{nm}}\right|_{\mathrm{glob}}. \tag{30}
$$

The infinitesimal global transformations of gauge field can be written down in the explicit form

$$
\left.\frac{\delta\mathcal{A}_{\nu,kl}}{\delta\omega_{mn}}\right|_{\mathrm{glob}} = (\mathcal{A}_{\nu,kp}S^{mn}_{ql} - S^{mn}_{kp}\mathcal{A}_{\nu,ql})\,\eta^{pq}, \tag{31}
$$

[5] We do not concern for problems connected to the search for nontrivial classical solutions in the Euclidean space. These solutions imply the instanton amplitudes for the transitions between the vacua in the quantum theory in the Minkowskian space-time. In addition, in the case under consideration the "duality" is written down for the group indices, while there is a possibility for the introduction of terms in the form of $\epsilon^{\mu\nu\lambda\gamma}\mathrm{Tr}[\mathcal{F}_{\mu\nu}\mathcal{F}_{\lambda\gamma}]$. Another note is that the group $SL(2,\mathbf{C})$ contains the subgroup $SU(2)$, so that the construction of classical solutions could be done by means of simple generalizing the case of $SU(2)$, though in this way one should investigate possibilities for a non-invariance of solutions, since additional transformations in the group of $SL(2,\mathbf{C})$ could lead to a trivialization, i.e. a transformations of "winding number".

where the spin operator of vector particle is equal to

$$S^{mn}_{kl} = \delta^m_k \delta^n_l - \delta^m_l \delta^n_k. \tag{32}$$

The infinitesimal transformation of gauge field can be written down in the form of covariant derivative for the transformation parameter ω:

$$\delta_\omega \mathcal{A}_{\mu,kl} = [\nabla_\mu \omega]_{kl} = \partial_\mu \omega_{kl} + (\mathcal{A}_{\mu,kp} S^{mn}_{ql} - S^{mn}_{kp} \mathcal{A}_{\mu,ql})\, \eta^{pq}\, \omega_{mn}. \tag{33}$$

So, we arrive to the following form of conservation law for the spinor current:

$$[\nabla_\mu\, j^\mu]^{mn} + \frac{1}{2}(T^{mn} - T^{nm}) = 0. \tag{34}$$

Introduce external sources $\mathcal{J}$ for the gauge field $\mathcal{A}$:

$$\mathcal{L}_\mathcal{J} = \mathcal{A}_{\mu,mn} \mathcal{J}^{\mu,mn}.$$

Let us fix the gauge in the Lorentz form, for example:

$$\partial^\mu \mathcal{A}_{\mu,mn} = 0, \tag{35}$$

and, following the ordinary procedure, add the gauge fixing term to the Lagrangian

$$\mathcal{L}_{\text{gf}} = \frac{1}{2\alpha}\,(\partial^\mu \mathcal{A}_{\mu,mn})^2. \tag{36}$$

The full Lagrangian for the gauge fields $\mathcal{L} = \mathcal{L}_\mathcal{A} + \mathcal{L}_{\text{gf}} + \mathcal{L}_\mathcal{J}$ remains invariant, if for any parameters of transformations ω we have

$$\left\{ \frac{\delta(\mathcal{L}_{\text{gf}} + \mathcal{L}_\mathcal{J})}{\delta\omega}\, \omega = 0, \quad \forall\omega \right\} \Rightarrow \quad \frac{\delta(\mathcal{L}_{\text{gf}} + \mathcal{L}_\mathcal{J})}{\delta\omega} = 0. \tag{37}$$

Let us calculate the operator in (37)

$$\frac{\delta(\mathcal{L}_{\text{gf}} + \mathcal{L}_\mathcal{J})}{\delta\omega}\, \omega = \frac{1}{\alpha}\,(\partial^\mu \mathcal{A}_\mu)\; \partial^\nu [\nabla_\nu \omega] + \mathcal{J}^\mu [\nabla_\mu \omega]. \tag{38}$$

Introduce the operator inverse to $\mathfrak{M}(\mathcal{A}) = \partial^\nu \nabla_\nu(\mathcal{A})$, and write down eq. (37) at $\omega = \mathfrak{M}^{-1}(\mathcal{A})\, \tilde{\omega}$, $\forall\tilde{\omega}$:

$$\frac{1}{\alpha}\,(\partial^\mu \mathcal{A}_\mu) + \mathcal{J}^\mu [\nabla_\mu(\mathcal{A})\, \mathfrak{M}^{-1}(\mathcal{A})] = 0. \tag{39}$$

Eq. (39) gives the identities of Slavnov–Taylor or the generalized identities of Ward–Takahashi. These equations are important in the quantum theory for the prove of renormalization [5]. They are usually written down for the partition functional dependent of sources $G[\mathcal{J}]$, so that the field is replaced by the variational derivative $\mathcal{A} \equiv \frac{\delta}{\delta\mathcal{J}}$:

$$\frac{1}{\alpha}\,\partial^\mu \frac{\delta G[\mathcal{J}]}{\delta \mathcal{J}^\mu} + \mathcal{J}^\mu \left[\nabla_\mu \left(\frac{\delta}{\delta\mathcal{J}} \right) \mathfrak{M}^{-1} \left(\frac{\delta}{\delta\mathcal{J}} \right) \right] G[\mathcal{J}] = 0. \tag{40}$$

Thus, the identities of Slavnov–Taylor show, which constraints[6] appear for the Green functions due to the independence of observables on the gauge degrees of freedom. In the quantum theory the partition functional is written down in the form of continual integral

$$e^{iG[\mathcal{J}]} = \int e^{i\int d^4x\,(\mathcal{L}_\mathcal{A}+\mathcal{L}_{\rm gf}+\mathcal{L}_\mathcal{J})} \det[\mathfrak{M}]\,\mathcal{D}\mathcal{A},$$

whereas one usually introduces the Grassmann ghosts by Faddeev–Popov

$$\det[\mathfrak{M}] = \int e^{i\int d^4x\,\mathcal{L}_{\rm gh}}\,\mathcal{D}c\,\mathcal{D}\bar{c},$$

where

$$\mathcal{L}_{\rm gh} = \bar{c}(x)\,\partial^\mu \nabla_\mu(\mathcal{A})\,c(x),$$

so that the operator $\mathfrak{M}^{-1}(\mathcal{A})$ is the propagator of ghosts c_{mn} in the external field $\mathcal{A}$.

3.2 Einstein gravity

Assume that there is a potential in the effective action, which leads to a nonzero vacuum strength of gauge fields. We suggest that the strength tensor can be represented by a decomposition in terms of sum over the tensors of some components, so that

$$\mathcal{F}_{\mu\nu,mn} = \mathcal{R}^{0[a]}_{\mu\nu,mn} + \mathcal{R}^{0[b]}_{\mu\nu,mn} + \frac{1}{2}\mathcal{R}_{\mu\nu,mn}(\Gamma), \tag{41}$$

so that the corresponding spin-connection components are determined by expressions with the following kinds of Lorentz structures:

$$\begin{aligned}
\mathcal{A}^{[a]}_{\mu,mn} &= \frac{1}{\sqrt{2}}\,\epsilon_{\mu\nu mn}a^\nu, \\
\mathcal{A}^{[b]}_{\mu,mn} &= \frac{1}{\sqrt{2}}\,(\eta_{\mu m}b_n - \eta_{\mu n}b_m), \\
\mathcal{A}^{[\Gamma]}_{\mu,mn} &= \frac{1}{2}\,\Gamma_{\mu,mn},
\end{aligned} \tag{42}$$

where the vacuum fields a and b have the expectation values

$$\begin{aligned}
\langle a^\mu\rangle = 0, \quad \langle b^\mu\rangle = 0, \qquad & \langle a^\mu a^\nu\rangle = \tfrac{1}{4}\,\eta^{\mu\nu} g^2_{\rm Pl} v_a^2, \\
\langle a^\mu b^\nu\rangle = \tfrac{1}{4}\,\eta^{\mu\nu} g^2_{\rm Pl} v_a\, v_b, \qquad & \langle b^\mu b^\nu\rangle = \tfrac{1}{4}\,\eta^{\mu\nu} g^2_{\rm Pl} v_b^2.
\end{aligned} \tag{43}$$

Thus, we have introduced two parallel time-like vectors, so that the corresponding connections are dual over the group indices: $\mathcal{A}^{[b]}_{\mu,mn} = -\frac{1}{2}\epsilon_{mnkl}\,\mathcal{A}_{[a]\mu}^{\;\;kl}$.

Let us emphasize that the prescriptions in (42) determine the components of strength tensor, which is valid under a gauge condition. Of course, we can apply the general gauge transformations to the components in (42), so that due to the linear decomposition in (41) the components of strength tensor are transformed independently. This fact implies that we deal with the presentation, where the gauge symmetry is not broken, while the strength tensor of

[6] In the quantum theory these identities can be broken in the case of anomaly.

gauge fields acquires the nontrivial vacuum expectation, that implies the spontaneous breaking of gauge symmtery.

Then, for the strength tensor of vacuum we have got

$$\begin{aligned}\mathcal{R}^{0[a]}_{\mu\nu,mn}(a) &= -(\epsilon_{\mu\alpha mk}\epsilon_{\nu\beta nk}-\epsilon_{\mu\alpha nk}\epsilon_{\nu\beta mk})a^\alpha a^\beta \\ &= a^2\eta_{\mu m}\eta_{\nu n}-a^2\eta_{\nu m}\eta_{\mu n}-a_\nu a_n\eta_{\mu m}+a_\nu a_m\eta_{\mu n}+a_\mu a_n\eta_{\nu m}-a_\mu a_m\eta_{\nu n}, \\ \mathcal{R}^{0[b]}_{\mu\nu,mn}(b) &= -\mathcal{R}^{0[a]}_{\mu\nu,mn}(b).\end{aligned} \tag{44}$$

The term of Lagrangian linear over the background strength tensor and dynamical tensor of curvature is equal to

$$\begin{aligned}\mathcal{L}_G &= \frac{1}{g^2_{\rm Pl}}\,\mathcal{R}_{\mu\nu}\cdot\left\langle\left(\tfrac{1}{2}a^2\eta^{\mu\nu}-a^\mu a^\nu\right)-\left(\tfrac{1}{2}b^2\eta^{\mu\nu}-b^\mu b^\nu\right)\right\rangle \\ &= \frac{1}{g^2_{\rm Pl}}\,\left(\tfrac{1}{2}\mathcal{R}\,\eta_{\mu\nu}-\mathcal{R}_{\mu\nu}\right)\,\langle a^\mu a^\nu-b^\mu b^\nu\rangle=-\tfrac{1}{4}\,(v_b^2-v_a^2)\,\mathcal{R}(\Gamma),\end{aligned} \tag{45}$$

where we have introduced the Ricci tensor $\mathcal{R}_{\mu\nu}(\Gamma)=\mathcal{R}_{\gamma\mu mn}(\Gamma)\,h^{m\gamma}\,h_\nu{}^n$ and the scalar curvature $\mathcal{R}(\Gamma)=\mathcal{R}_{\mu\nu}\,\eta^{\mu\nu}$. Sure, the Lagrangian of (45) strictly coincides with the Lagrangian of Einstein–Hilbert gravity in the theory of general relativity, if we denote the gravitational constant $\kappa=\frac{1}{v_b^2-v_a^2}$ and add the factor defining the invariant measure $\det[h]\,d^4x$ depending on the tetrad $h_{m\mu}$.

Here we challenge the role of tetrad. We have introduced the tetrad as an auxiliary field, which has been locally and globally reduced to the unit symbol of Kronecker in the Minkowskian space-time. Under the gauge field condensation implying the spontaneous breaking of gauge symmtery, the nontrivial vacuum configuration leads to that the tetrad becomes a field depending on the coordinates of space-time, so that it is only locally reduced to the unit under the action of general coordinate transformations, while we explore the linear approximation of (45), and the dynamical characteristics of connection Γ can be ordinary reassigned to the characteristics of tetrad or to the world metric tensor constructed by the tetrad.

Indeed, following Palatini [6], we see that the Einstein–Hilbert Lagrangian composed by the Ricci tensor, depending on the connection[7], and the auxiliary field of metric tensor[8], which is ordinary defined as a quadratic form of tetrad. Then the variation of action over the connection leads to the equations of motion giving the constraints for the covariant derivative of metric tensor, so that this derivative is equal to zero. This fact implies that the connection is consistent with the metrics, and it is expressed in terms of Christoffel symbols. The action is independent of auxiliary field, that leads to the Einstein–Hilbert equations in the theory of general relativity, so that the field of metric tensor determining the connection brings the dynamical characteristics of connection. The procedure of canonical quantization of Hamiltonian dynamics shows that for this field we have two dynamical massless modes with the helicity equal to ± 2 [9].

Thus, the vacuum expectation values for the fields a and b defined above are related with the Planck mass $m^2_{\rm Pl}=v_b^2-v_a^2$, so that in the case of comparable values[9] of v_a and v_b at

[7] In this action we concern for the symmetric connection after the transition to the world indices (see [7]).

[8] We follow the presentation given in [8].

[9] We argue for this assumption below.

the energies of dynamical fields less than the Planck scale we can neglect the contributions quadratic over the strength tensor in comparison with the linear Einstein–Hilbert Lagrangian of (45). Generally, at $v_b < v_a$ we could get the negative gravitational constant, i.e. anti-gravity.

Then, we arrive to the Einstein–Hilbert theory of gravity as the low-energy limit of full Lagrangian with the gauge field condensation, *viz.* the spontaneous breaking of gauge symmtery, so that the vacuum fields lead to the curved space-time.

3.3 Cosmological term

Let us consider the term quadratic over the background vacuum strength in the action of spin-connection. The corresponding contribution to the Lagrangian determines the density of vacuum energy due to the background connection. We can easily show that it is equal to

$$\mathcal{L}_{[\mathcal{R}^0]^2} = 3\, g^2_{\rm Pl}\, (v_b^2 - v_a^2)^2 = 3\, g^2_{\rm Pl}\, m^4_{\rm Pl}, \tag{46}$$

i.e. we get a huge cosmological constant[10]. However, this contribution could be cancelled by the term determining the interaction of vacuum field with the spinor current, if there is a corresponding vacuum condensate of spinor field. Indeed, introduce the vacuum field of spinor current

$$j_0^{\mu,mn} = \frac{1}{\sqrt{2}}\,\zeta\, \epsilon^{\mu\nu mn} a_\nu \frac{(v_b^2 - v_a^2)^2}{v_a^2}, \tag{47}$$

where ζ is a dimensionless factor, and calculate the product of current and the vacuum connection in (42), where the component of connection with the field a contributes only, so that

$$\mathcal{L}_{j\mathcal{R}^0} = -3\,\zeta\, g^2_{\rm Pl}\, (v_b^2 - v_a^2)^2. \tag{48}$$

We see that the cosmological constant is cancelled in the sum of quadratic vacuum curvature (46) and contribution from the interaction of vacuum spinor and gauge fields (48), if $\zeta = 1$.

Sure, the correlation of vacuum gauge and spinor fields, i.e. "fine tuning" necessary for the strict cancellation of cosmological constant, points to some physical reasons, which nature can be caused by the supersymmetry, for instance, though that requires an additional study beyond the scope of this work. We will make the only remark on this problem in section 5. Here we point out that the strict cancellation of cosmological term can be broken by small fluctuations of vacuum fields, so that the balance state of system would be destroyed, and at the appropriate sign of cosmological constant producing the repulsion, for example, the inflation expansion of universe would happen [10]. In this way, there is a problem of border regions, where one should match global fluctuations different by their value or, probably, sign, so that the corresponding domain walls appear not only between two expanding universes, but also between contracting and expanding ones.

Thus, we draw the conclusion that the cancellation of cosmological constant can take place, while the mechanism is not clear enough and should be investigated, and the fluctuations of vacuum fields result in cosmological implications.

[10] It would be quite naively to think that the expression for the density of vacuum energy in terms of fields a and b in eq. (46) is the form of potential V, if we put $b^2 - a^2 = s^2$, then $V \sim -s^4$ and it is not restricted from below. Indeed, in the quantum chromodynamics, for example, the density of vacuum energy is negative and proportional to the gluon condensate, so that vacuum energy infinitely drops with the increase of gluon condensate. The expression for the density of energy in terms of vacuum fields is generally determined by the dimensional analysis, and it does not contain a full information on the nonlinear dynamics, wherein the vacuum expectation values are restricted, and there is the anomaly in the trace of energy-momentum tensor.

3.4 Massive modes

Consider excitations on the background of vacuum field a, i.e. in eq. (42) we substitute for a^μ by the field

$$a^\mu + \frac{g_{\rm Pl}}{\sqrt{2}}\,\omega^\mu(x).$$

Then the term of Lagrangian quadratic in ω^μ has the form

$$\mathcal{L}_\omega \;=\; -\tfrac{1}{2}\left((\partial^\mu\omega_\nu)^2 + \tfrac{1}{2}(\partial^\mu\omega_\mu)^2\right) + 6\,g_{\rm Pl}^2\cdot[3v_a^2 - v_b^2]\,\omega_\mu^2, \tag{49}$$

where the term $(\partial^\mu\omega_\mu)^2$ corresponds to the gauge condition, while the mass of vector field ω_μ is determined by the equality

$$m_\omega^2 = 12\,g_{\rm Pl}^2\,(2v_a^2 - m_{\rm Pl}^2),$$

so that the necessary constraint for the positive definiteness of both the square of ω^μ mass and the gravitation constant is the inequality

$$\frac{1}{3}\,v_b^2 < v_a^2 < v_b^2.$$

Particularly, the situation of $v_b^2 - v_a^2 = m_{\rm Pl}^2 \ll v_a^2 \sim m_{\rm Pl}^2/g_{\rm Pl}^2$ is of interest. In this case the vector field has the mass close to the Planck scale.

Further, let us consider the possibility for a variation of absolute value for b and introduce

$$b^\mu\tilde{\phi},$$

in (42). Then the term of Lagrangian quadratic in $\tilde{\phi}$ is equal to

$$\mathcal{L}_\phi = \frac{9}{8}v_b^2(\partial_\mu\tilde{\phi})^2 - 6\,g_{\rm Pl}^2\,v_a^2 v_b^2\,\tilde{\phi}^2 = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{8}{3}\,g_{\rm Pl}^2\,v_a^2\,\phi^2, \tag{50}$$

where we have denoted $\phi = \frac{3}{2}v_b\,\tilde{\phi}$, and its mass equals

$$m_\phi^2 = \frac{16}{3}\,g_{\rm Pl}^2\,v_a^2.$$

We see that the mass of scalar field is also at the Planck scale. The above consideration on the mass of ϕ is valid in the vicinity of $\phi \to 0$. However, we study the situation, when $\langle\phi\rangle = \frac{3}{2}v_b$, i.e. the field is, probably, displaced from its local minimum, that is an ordinary scenario with the inflation [10].

Summarizing the consideration of massive modes and gravitation Lagrangian, we describe 3 massive modes of vector field, 1 massive scalar field and 2 polarizations of massless graviton, i.e. 6 degrees of freedom, while in the standard approach to the gauge theory we should expect that the dynamics involves 6 kinds of massless particles with two transversal polarizations because of 6 generators of group transformations, i.e. 12 physical modes. In addition, we can easily see that the introduction of scalar field for the scale variation of a analogous to ϕ as well as the vector field linear to b similar to ω^μ would result in negative kinetic energies for these additional fields. This fact challenges a more accurate and strict consideration of spin-connection dynamics, that will be the subject of next section.

3.5 Hamiltonian dynamics

Let us consider the standard approach for the description of gauge field dynamics. In this way, the field $\mathcal{A}_{0,mn}$ has no derivative with respect to time in the Lagrangian, and it is not a dynamical variable. This field is a Lagrange factor, so that we can put

$$\mathcal{A}_{0,mn} = 0. \tag{51}$$

Introduce the notations for the components of strength tensor

$$\begin{aligned} \mathcal{F}_{0k,0n} = \mathcal{E}_{k,n}, \qquad \mathcal{F}_{0k,ij} \cdot \tfrac{1}{2}\, \epsilon^{ijn} = \tilde{\mathcal{E}}_{k}{}^{n}, \\ \mathcal{F}_{lp,0n} \cdot \tfrac{1}{2} \epsilon^{lpk} = \mathcal{H}^{k}{}_{n}, \qquad \mathcal{F}_{lp,ij} \cdot \tfrac{1}{2}\, \epsilon^{ijn}\, \tfrac{1}{2} \epsilon^{lpk} = \tilde{\mathcal{H}}^{k,n}, \end{aligned} \tag{52}$$

where the indices are running in the limits $(1,2,3)$, and $\mathcal{E}$ is an electric field, $\tilde{\mathcal{E}}$ is a pseudo-electric field, $\mathcal{H}$ is a magnetic field, $\tilde{\mathcal{H}}$ is a pseudomagnetic field. Taking into account (51), we get

$$\mathcal{F}_{0i,mn} = \partial_0\, \mathcal{A}_{i,mn}, \tag{53}$$

so that the electric fields are given by the velocities of gauge fields.

Calculating the trace for the square of strength tensor and the product of strength tensor with the tensor dual over the space-time world indices,

$$\mathcal{F}^{D}_{\mu\nu,mn} = \frac{1}{2}\, \epsilon_{\mu\nu}{}^{\alpha\beta}\, \mathcal{F}_{\alpha\beta,mn}, \qquad \mathcal{F}_{\mu\nu,mn} = -\frac{1}{2}\, \epsilon_{\mu\nu}{}^{\alpha\beta}\, \mathcal{F}^{D}_{\alpha\beta,mn}, \tag{54}$$

we find the following expressions for the field invariants, which are scalar and do not depend on the gauge:

$$\begin{aligned} I_1 &= \mathcal{E} \cdot \mathcal{H} - \tilde{\mathcal{E}} \cdot \tilde{\mathcal{H}}, \\ I_2 &= \mathcal{E} \cdot \tilde{\mathcal{E}} - \mathcal{H} \cdot \tilde{\mathcal{H}}, \\ I_3 &= \mathcal{E} \cdot \tilde{\mathcal{H}} + \tilde{\mathcal{E}} \cdot \mathcal{H}, \end{aligned} \tag{55}$$

along with the Lagrangian of gauge field

$$\mathcal{L} = \frac{1}{2 g_{\rm Pl}^2} \left[\mathcal{E}^2 - \tilde{\mathcal{E}}^2 - \mathcal{H}^2 + \tilde{\mathcal{H}}^2 \right], \tag{56}$$

where the scalar products are contracted over the three-dimensional Euclidean indices in the group and space.

In the formalism under consideration the momenta of fields coincide with the electric fields. The Hamiltonian of system can be written down in the form

$$\mathsf{H} = \frac{1}{2 g_{\rm Pl}^2} \left[\mathcal{E}^2 - \tilde{\mathcal{E}}^2 + \mathcal{H}^2 - \tilde{\mathcal{H}}^2 \right], \tag{57}$$

so that introducing the gauge

$$\partial_i\, \mathcal{A}_{i,mn} = 0, \tag{58}$$

we can calculate its Poisson bracket with the Hamiltonian of (57), i.e. the time derivative of constraint, and get new constraint (the Gauss law)

$$\partial_i\, \partial_0\, \mathcal{A}_{i,mn} = 0, \;\; \Leftrightarrow \;\; \partial_i\, \mathcal{E}_{i,n} = 0, \quad \partial_i\, \tilde{\mathcal{E}}_{i,n} = 0. \tag{59}$$

Thus, Hamiltonian (57) depends on the transverse components of electric fields as well as the magnetic fields expressed in terms of transverse gauge fields, so that

$$\mathsf{H} = \frac{1}{2g_{\rm Pl}^2}\,\Big[\mathcal{E}_\perp^2(\mathcal{A}_\perp) - \tilde{\mathcal{E}}_\perp^2(\mathcal{A}_\perp) + \mathcal{H}^2(\mathcal{A}_\perp) - \tilde{\mathcal{H}}^2(\mathcal{A}_\perp)\Big]. \tag{60}$$

We know one exact solution of field equations following from the Hamiltonian of (60): all of the fields and momenta are equal to zero. However, we see in (60) that such the Hamiltonian results in the instability of zero solution, and the perturbation theory built in the vicinity of zero includes the modes with the negative kinetic energy (pseudoelectric field). Nevertheless, this fact does not imply that there is no stable state in the theory since the nonlinear character of field equations can generally lead to a restriction of total energy from below. It is evident that in the case of existing the stable solution it is determined by dynamical fields, which kinetic energy could differ from zero.

A possibility of such situation can be shown in the following way. For the fields determined by the dual strength tensor we have the relations

$$\tilde{\mathcal{E}}_k{}^n = -[\tilde{\mathcal{H}}^D]_k{}^n, \quad \tilde{\mathcal{H}}^{kn} = -[\tilde{\mathcal{E}}^D]^{kn}. \tag{61}$$

Therefore the Lagrangian of (56) can be rewritten in the form

$$\mathcal{L} = \frac{1}{2g_{\rm Pl}^2}\,\Big[\mathcal{E}^2 - [\tilde{\mathcal{H}}^D]^2 - \mathcal{H}^2 + [\tilde{\mathcal{E}}^D]^2\Big], \tag{62}$$

so that eq. (62) leads to the Hamiltonian

$$\mathsf{H} = \frac{1}{2g_{\rm Pl}^2}\,\Big[\mathcal{E}_\perp^2(\mathcal{A}_\perp) + [\tilde{\mathcal{E}}_\perp^D]^2(\tilde{\mathcal{A}}_\perp^D) + \mathcal{H}^2(\mathcal{A}_\perp, \tilde{\mathcal{A}}_\perp^D) + [\tilde{\mathcal{H}}^D]^2(\mathcal{A}_\perp, \tilde{\mathcal{A}}_\perp^D)\Big], \tag{63}$$

where we have introduced the fields[11]

$$\begin{aligned}
&\mathcal{A}_{\perp i,n} = \mathcal{A}_{i,0n}, && \partial_i\,\mathcal{A}_{i,0n} = 0,\\
&\tilde{\mathcal{A}}^D_{\perp i,n} = \tfrac{1}{2}\,\epsilon_{kln}\mathcal{A}^D_{i,kl}, && \partial_i\,\mathcal{A}^D_{i,kl} = 0,\\
&\partial_i\mathcal{E}_{\perp i,n} = 0, && \partial_i\tilde{\mathcal{E}}^D_{\perp i,n} = 0,\\
&\mathcal{E}_{\perp i,n} = \partial_0\,\mathcal{A}_{\perp i,0n}, && \tilde{\mathcal{E}}^D_{\perp i,n} = \tfrac{1}{2}\,\epsilon_{kln}\,\partial_0\,\mathcal{A}^D_{\perp i,kl},
\end{aligned} \tag{64}$$

under the condition that the gauge field allows such the separation of variables, of course, and we can believe that putting the Lagrange factors equal to zero is consistent

$$\mathcal{A}_{0,mn} = 0, \quad \tilde{\mathcal{A}}^D_{0,mn} = 0.$$

[11]Under the conditions of transversity for the fields of (64) we can show that the Poisson bracket $\{\tilde{\mathcal{H}}, \mathcal{E}\} = -\{\tilde{\mathcal{E}}^D, \mathcal{E}\} \approx 0$ on the surface of constraints, i.e. the momenta of fields $\mathcal{A}$ and $\tilde{\mathcal{A}}^D$ commute. The fact that the fields $\tilde{\mathcal{H}}^D$ and $\mathcal{H}$ do not depend on the time derivatives, follows from the equations determining the exclusion of fields $\tilde{\mathcal{A}}$ and $\mathcal{A}^D$:

$$\left\{\begin{array}{lcl} \partial_0\,\tilde{\mathcal{A}} &=& -\tilde{\mathcal{H}}^D(\mathcal{A}^D, \tilde{\mathcal{A}}^D),\\ \partial_0\,\mathcal{A}^D &=& -\mathcal{H}(\mathcal{A}, \tilde{\mathcal{A}}). \end{array}\right.$$

Thus, it is possible that the dependence of magnetic fields on $\mathcal{A}$ and $\tilde{\mathcal{A}}^D$ becomes nonlocal in time (this should not break the causality, since we transform the theory, wherein the causality principle is valid). However, we see that the expression of Lagrangian in terms of non-homogeneous fields $\mathcal{A}$ and $\tilde{\mathcal{A}}^D$ generally leads to the problem on the separation of variables.

The expression for the Hamiltonian in (63) shows its positive definiteness. However, the construction of perturbation theory under this Hamiltonian is problematic since we assume the implicit substitution of particular gauge fields in terms of dual field, while we must make explicit substitutions in the nonabelian vertices of interactions. Nevertheless, we suggest that the operation with the dual fields allows us to present some characteristic features of spin-connection gauge field[12].

3.5.1 Dual ansatz

Following the notations of (42) in section 3.2, introduce the dual strength tensor as a sum over the tensors for the components of connection $\mathcal{A}^{\mathcal{D}}$, so that

$$\mathcal{F}^D_{\mu\nu,mn} = \mathcal{R}^{0[a^D]}_{\mu\nu,mn} + \mathcal{R}^{0[b^D]}_{\mu\nu,mn} + \frac{1}{2}\mathcal{R}_{\mu\nu,mn}(\Gamma^D), \tag{65}$$

and

$$\begin{aligned} \mathcal{A}^{[b^D]}_{\mu,mn} &= \frac{1}{\sqrt{2}}\,\epsilon_{\mu\nu mn}[b^D]^\nu, \\ \mathcal{A}^{[a^D]}_{\mu,mn} &= \frac{1}{\sqrt{2}}\,(\eta_{\mu m}a^D_n - \eta_{\mu n}a^D_m), \qquad (66) \\ \mathcal{A}^{[\Gamma^D]}_{\mu,mn} &= \frac{1}{2}\,\Gamma^D_{\mu,mn}, \qquad (67) \end{aligned}$$

where the vacuum fields a^D and b^D have the expectation values

$$\begin{array}{rr} \langle a^D_\mu\rangle = 0, \quad \langle b^D_\mu\rangle = 0, & \langle a^D_\mu a^D_\nu\rangle = \frac{1}{4}\,\eta_{\mu\nu}g^2_{\rm Pl}[v^D_a]^2, \\ \langle a^D_\mu b^D_\nu\rangle = \frac{1}{4}\,\eta_{\mu\nu}g^2_{\rm Pl}v^D_a\, v^D_b, & \langle b^D_\mu b^D_\nu\rangle = \frac{1}{4}\,\eta_{\mu\nu}g^2_{\rm Pl}[v^D_b]^2. \end{array} \tag{68}$$

Then, we have the following contribution to the vacuum strength tensor:

$$\begin{aligned} -\tfrac{1}{2}\,\epsilon_{\mu\nu}{}^{\alpha\beta}\,\mathcal{R}^{0[b^D]}_{\alpha\beta,mn}(b^D) &= -(b^2_D\,\epsilon_{\mu\nu mn} - \epsilon_{\mu\nu m\alpha}\,b^\alpha_D b^D_n + \epsilon_{\mu\nu n\alpha}\,b^\alpha_D b^D_m), \\ \mathcal{R}^{0[a^D]}_{\mu\nu,mn}(a^D) &= -\mathcal{R}^{0[b^D]}_{\mu\nu,mn}(a^D). \end{aligned} \tag{69}$$

Noting

$$\mathcal{F}^2 = -\mathcal{F}^2_D,$$

we see that the term of Lagrangian linear over the background strength tensor and the dynamical tensor of curvature is equal to[13]

$$\mathcal{L}_G = -\tfrac{1}{4}\,(v_b^2 - v_a^2)\,\mathcal{R}(\Gamma) - \tfrac{1}{4}\,([v^D_b]^2 - [v^D_a]^2)\,\mathcal{R}(\Gamma^D). \tag{70}$$

It is evident that the motion equations for the connections Γ and Γ^D lead to zero covariant derivatives of metric tensor over both connections, so that in the linear limit over the

[12]Sure, in this construction we suppose the assumption on a possibility of correct separating the variables.

[13]We do not consider terms in the form of $(v_a^2 - v_b^2)\epsilon^{\mu\nu mn}\mathcal{R}_{\mu\nu mn}(\Gamma^D)$ as well as that of dual to it, since such contributions are equal to zero for the symmetric connection with the indices on the world coordinates.

strength of dynamical connection-fields the massless modes are coherent, and the connections are expressed by the symbols of Christoffel, $\Gamma^D = \Gamma$ and $R(\Gamma^D) = R(\Gamma)$. Therefore

$$\mathcal{L}_G = -\frac{1}{4}\,\left(v_b^2 - v_a^2 + [v_b^D]^2 - [v_a^D]^2\right)\,\mathcal{R}(\Gamma). \tag{71}$$

It is interesting to note that the cosmological constant determined by the vacuum fields in the strength tensor and dual one takes the form

$$\mathcal{L}_{[\mathcal{R}^0]^2} = 3\,g_{\rm Pl}^2\,[(v_b^2 - v_a^2)^2 - ([v_b^D]^2 - [v_a^D]^2)^2], \tag{72}$$

and it is cancelled, if[14]

$$|[v_b^D]^2 - [v_a^D]^2| = |v_b^2 - v_a^2| = \frac{1}{2}\,m_{\rm Pl}^2. \tag{73}$$

Thus, the cancellation of cosmological term in this mechanism does not require an introduction of spinor condensate, but we can again expect that fluctuation of vacuum fields probably could lead to the expanding or contracting universe.

The expectation value of vacuum strength is given by the expression

$$\langle \mathcal{F}_{\mu\nu,mn}\rangle = \tfrac{1}{2}\,(\eta_{\mu m}\eta_{\nu n} - \eta_{\mu n}\eta_{\nu m})\,(v_a^2 - v_b^2) - \tfrac{1}{2}\,\epsilon_{\mu\nu mn}([v_b^D]^2 - [v_a^D]^2), \tag{74}$$

so that the expectation values of electric and magnetic fields[15] are equal to

$$\begin{aligned} \langle \mathcal{E}_{m,n}\rangle &= \tfrac{1}{2}\,(v_a^2 - v_b^2)\delta_{mn}, \qquad \langle \mathcal{H}_{m,n}\rangle = \tfrac{1}{2}\,([v_a^D]^2 - [v_b^D]^2)\delta_{mn},\\ \langle \tilde{\mathcal{H}}_{m,n}\rangle &= \tfrac{1}{2}\,(v_a^2 - v_b^2)\delta_{mn}, \qquad \langle \tilde{\mathcal{E}}_{m,n}\rangle = \tfrac{1}{2}\,([v_a^D]^2 - [v_b^D]^2)\delta_{mn}. \end{aligned} \tag{75}$$

The consideration of massive modes is analogous to section 3.4, and it leads to massive vector and scalar fields[16], so that

$$\begin{aligned} m_\omega^2 &= 12\,g_{\rm Pl}^2\,\left(2v_a^2 - \tfrac{1}{2}\,m_{\rm Pl}^2\right), \qquad & m_\phi^2 &= \tfrac{16}{3}\,g_{\rm Pl}^2\,v_a^2,\\ m_{\omega_D}^2 &= 12\,g_{\rm Pl}^2\,\left(2[v_a^D]^2 - \tfrac{1}{2}\,m_{\rm Pl}^2\right), \qquad & m_{\phi_D}^2 &= \tfrac{16}{3}\,g_{\rm Pl}^2\,[v_a^D]^2. \end{aligned} \tag{76}$$

We see that the number of gauge constraints for the fields is equal to 6 because[17]

$$\begin{aligned} a^2 - v_a^2 = 0, &\qquad b^2 - v_b^2 = 0,\\ \partial^\mu\,\omega_\mu = 0 &\qquad \tilde{\omega}_\mu - b_\mu\,(b\cdot\tilde{\omega}) = 0, \end{aligned}$$

while the analogous constraints for the fields determining the dual strength tensor probably represent the additional conditions for the momenta of gauge fields, since in the dual notations they are transformed in some ordinary constraints for the fields.

Thus, we have got 12 dynamical fields: 3 polarizations of massive vector field and 3 ones dual to them, the scalar massive field and that of dual to it, as well as 2 modes of massless field with the spin 2 and 2 dual modes, which are coherent in the limit of Einstein gravity, since the massless modes differ at the level of corrections suppressed by the square of ratio given by the energy of field quanta over the Planck mass.

[14] In comparison with section 3.2, the relation between the difference of vacuum expectation values and the Planck mass changes.

[15] Calculating the squares of fields entering the Lagrangian, we take into account their nonzero dispersion.

[16] The cross-terms linear in the massive fields and dual strength tensor of vacuum fields do not give new effects: the gauge of vector fields takes a general form of $\partial^\mu\,\omega_\mu - \Phi(a^d, b^d) = 0$, and for the scalar fields the terms three-linear in the vacuum field are equal to zero.

[17] We put $\tilde{\omega}_\mu = b_\mu\tilde{\phi}$.

3.5.2 Vacuum fields and equations of motion

In the previous sections we do not address the problem on how one could actually show an effective action providing the above choice of background vacuum fields, otherwise the prescriptions (41), (42) look rather random. Of course, such the presentation of effective action would imply an exact or, at the very least, approximate solution of field equations in a manifest form, that seems to be quite a difficult task as concerns for nonabelian theories. The similar situation is observed in QCD, where we know that the gauge field has a vacuum condensate involving an infrared nonperturbative dynamics, however, we cannot prove or derive a form of effective action except its modelling. In the theory under study we assume the appearance of gauge field condensation, in general, and provide the expressions modelling this phenomenon. The form of vacuum field ansatz is quite transparent from the Lorentz-structure point of view. However, the formulae cannot be proved to the moment.

In this section we investigate, how the vacuum fields enter the field equations. First, we suggest that the vacuum expectation of strength tensor presented above is valid in the Fock-Schwinger gauge of fixed point

$$x^\mu \cdot \mathcal{A}_{\mu,mn}(x) = 0,$$

that implies the introduction of special marked point in the configuration space. In this gauge the field can be expressed in terms of the strength tensor

$$\mathcal{A}_{\mu,mn}(x) = \int\limits_0^1 dz\, z\, x^\nu \cdot \mathcal{F}_{\nu\mu,mn}(z\cdot x), \tag{77}$$

and we put the vacuum expectation $\langle \mathcal{A}_{\mu,mn}(x)\rangle$ by making use of (74) in the right-hand side of (77) and integrating out since the vacuum strength tensor is independent of x. However, the form (77) of Fock-Schwinger gauge in the nonabelian theory is not consistent with the global vacuum expectation of strength tensor. Formally, we could joint (74) and (77) with no contradiction, i.e. we could reproduce the strength tensor by its definition under the substitution of (77) if we ignore the totally antisymmetric tensor $\epsilon_{\mu\nu\alpha\beta}$ multiplied by the mixed product of vacuum expectations for the gauge field and dual to it and suppose (73), that implies the cancellation of cosmological term. Moreover, the vacuum strength tensor under the condition of (73) is anti-selfdual, and hence, it satisfies the field equations with no sources. This selfduality, as well known, results in the local minimization of gauge field action.

In detail, the field equations are given by

$$[\nabla^\mu(\mathcal{A})\mathcal{F}_{\mu\nu}(x)]_{mn} = j_{\nu,mn}(x),$$

where the covariant derivative is defined in section 3.1. For the vacuum field without a dispersion we put

$$[\nabla^\mu(\langle\mathcal{A}\rangle)\ \langle\mathcal{F}_{\mu\nu}(x)\rangle]_{mn} = \langle j_{\nu,mn}(x)\rangle,$$

which under (77) straightforwardly results in

$$\begin{aligned}\langle j^{\alpha,mn}\rangle &= \frac{1}{2}\Big((v_a^2 - v_b^2)^2 - \big([v_a^D]^2 - [v_b^D]^2\big)^2\Big)\,(g^{m\beta}g^{n\alpha} - g^{m\alpha}g^{n\beta})x_\beta \\ &- (v_a^2 - v_b^2)([v_a^D]^2 - [v_b^D]^2)\epsilon^{mn\alpha\beta}x_\beta.\end{aligned} \tag{78}$$

As we have already mentioned, the first term in the above equation disappears under the anti-selfduality and the cancellation of cosmological constant, while the second is the artefact of (77), and it should be ignored, which is evident since the selfduality of strength tensor guarantees that the source current is equal to zero.

Therefore, the field equations are satisfied until

1. the cosmological constant is put to zero;
2. the spinor current is equal to zero.

Thus, we have shown how the offered vacuum condensates can enter the field equations and be related with the physical contents of the model under consideration.

4 Dirac spinors

The Dirac bispinors

$$\psi = \begin{pmatrix} \theta_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix},$$

are operated by the algebra of γ-matrices, which are defined by the following relations:

$$\gamma^m = \rho_+ \otimes \sigma^m + \rho_- \otimes \bar{\sigma}^m, \tag{79}$$

so that

$$\{\gamma^m, \gamma^n\} = 2\,\eta^{mn}, \tag{80}$$

because $\rho_+ = \frac{1}{2}(\sigma_1 + i\,\sigma_2)$ is a raising operator, $\rho_- = \rho_+^\dagger$ and

$$\rho_+\rho_- + \rho_-\rho_+ = 1, \quad \rho_+^2 = 0. \tag{81}$$

Eq. (81) points to that the quantities $\rho_L = \rho_+\rho_-$, $\rho_R = \rho_-\rho_+$ are projectors. Then we find

$$\{\gamma^n, \gamma^m\} = \rho_+\rho_- \otimes \{\sigma^n\bar{\sigma}^m + \sigma^m\bar{\sigma}^n\} + \rho_-\rho_+ \otimes \{\bar{\sigma}^n\sigma^m + \bar{\sigma}^m\sigma^n\} = 2\,\eta^{nm}. \tag{82}$$

4.1 Internal symmetry

Since the Weyl factors enter the definitions of Dirac γ-matrices and bispinors, the above study of invariant representation for the σ-matrices remains valid. The additional subject is the invariant changes of representations for the matrices ρ:

$$\rho_+' = e^{i\lambda}\, f \cdot \rho_+ \cdot f^{-1}, \tag{83}$$

where $f \in SU(2)$, $\lambda \in \Re$ can be local functions. As the left-handed and right-handed spinors in the massless Dirac field, which get the mass due to the interaction with scalar particles, are independent and they can be separately put equal to zero, the additional group of symmetry for the Dirac spinors is the product of groups on the chiral fields, so that

$$U(1)_L \otimes SU(2)_L \otimes U(1)_R \otimes SU(2)_R. \tag{84}$$

This fact can be verified and confirmed by a straightforward calculation, if we write down the generators of unitary transformations for representations of Dirac spinors in the form of sum over 16 independent basis matrices and find the constraints for the coefficients of linear span from the equation for the conservation of Hamiltonian form for the Dirac field in the massless case.

The group of internal symmetry (84) includes the group of electroweak symmetry, which is $U(1)\otimes SU(2)_L$. We see that the combining of two Weyl spinors in the multiplet of bispinor involves the gauge symmetry for the Dirac spinors, so that, if we identify the group of electroweak symmetry with the subgroup of (84), then we expect that the partner of electron in the weak iso-doublet, i.e. neutrino, should be also the Dirac particle. A trivial speculation concerns for that the group of internal symmetry could be extended, if the multiplet is built by several Weyl spinors, and the group of quantum chromodynamics, for instance, does not contradict with the above investigation for the Dirac spinors. More interesting challenge is an observability of extension for the electroweak group, that follows from (84). So, the multiplet can be really more extended than the Dirac spinor, and hence, the additional group includes the subgroup $U(1)\otimes SU(2)_R$.

Thus, in contrast to the Weyl spinors, for the invariant representations of Dirac algebra on the bispinors there is the gauge group of internal symmetry (84) including the electroweak group.

5 Supersymmetry

In the study of possible mechanism for the cancellation of cosmological constant in section 3.3, the problem on the necessity of supersymmetry between the bosonic and fermionic fields has been mentioned.

In this section we discuss the problem on supersymmetry from a general point of view and offer a BRST-generalization of operator for the evolution of Weyl spinor, that leads to the supersymmetric transformations in the space of world coordinates and global spinors. Thus, the introduction of parametric transformations determined by global Grassmann variables for the Lagrangian of Weyl spinors results in the necessity of supersymmetry in the theory with the Weyl spinors.

For the Lagrangian of Weyl spinor

$$\mathcal{L} = \theta\sigma^\mu p_\mu\bar{\theta},$$

the partition function[18] W depending on sources η, $\bar{\eta}$, has the form

$$W(\eta,\bar{\eta}) = \int d\theta d\bar{\theta}\; e^{i\mathcal{L}+i\eta\theta+i\bar{\theta}\bar{\eta}} = N_W\cdot e^{iG} = N_W\cdot\exp\{-i\bar{\eta}\bar{K}\eta\},$$

where

$$\bar{K}\cdot\sigma^\mu p_\mu = 1,$$

[18] The partition functional of Green functions can be represented as the limit of partition function product: $\lim\limits_{M\to\infty}\prod\limits_{m=1}^{M}\int d\theta_m d\bar{\theta}_m\; e^{i\theta_m\sigma^\mu p_\mu\bar{\theta}_m dx_m+i\eta_m\theta_m dx_m+i\bar{\theta}_m\bar{\eta}_m dx_m}$, where we can redefine the fields $\theta_m=\theta(x_m)$ and $\eta_m=\eta(x_m)$, so that the element of space-time dx_m would enter in a implicit way, that allows us to make a transition to global spinors in a simple manner. Therefore, we can consider the partition function for our purpose.

so that the fields are equal to

$$\theta = -i\frac{1}{W}\frac{\partial W}{\partial \eta} = \frac{\partial G}{\partial \eta}, \quad \bar{\theta} = -i\frac{1}{W}\frac{\partial W}{\partial \bar{\eta}} = \frac{\partial G}{\partial \bar{\eta}}.$$

It is convenient to fix the normalization of fields so that $\mathcal{L}$ is the dimensionless quantity, and the Weyl spinor has the dimension of inverse square root of energy (we can easily reconstruct the usual normalization of fields and Lagrangian with no problem). In this way, the element of space-time measure is included into the normalization.

The operator of evolution has the form

$$U(d\tau) = e^{i\mathcal{L}d\tau},$$

where the field is given by the derivatives over the sources, and $W(U) = U \cdot W \cdot U^{\dagger}$. In the ordinary normalization we have $d\tau = Edt$, E is an energy of state, dt is a shift of time.

Under the action of operator U, the fields on the partition function G get the phase shift $d\tau$, reflecting the arbitrary choice of time reference-point on the trajectory of free Weyl spinor. Indeed, in accordance with the Hausdorff formula

$$e^A \cdot e^B = e^{A+B+[A,B]/2+\ldots}$$

we find

$$\delta\theta \cdot G = i[\mathcal{L}, -\bar{\eta}\bar{K}]d\tau \cdot G = -id\tau\, \theta \cdot G$$

so that $\theta(d\tau) = e^{-id\tau}\theta$.

The BRST-generalization of evolution operator can be obtained by fixing the shift in the form

$$d\hat{\tau} = 2i\left[\frac{\xi\theta}{\theta\theta} - \frac{\bar{\xi}\bar{\theta}}{\bar{\theta}\bar{\theta}}\right],$$

where ξ is an infinitesimal Weyl spinor, the parameter of transformation, so that $\{\xi, \theta\} = 0$ and

$$\delta_\xi = i\mathcal{L}d\hat{\tau} = -\xi\sigma^\mu p_\mu\bar{\theta} + \theta\sigma^\mu p_\mu\bar{\xi}.$$

Then we find that the action of operator on the fields gives

$$\delta_\xi\theta \cdot G = [\delta_\xi, \frac{\partial G}{\partial \eta}] \cdot G = [\delta_\xi, -\bar{\eta}\bar{K}] \cdot G = \xi \cdot G, \tag{85}$$

$$\delta_\xi\bar{\theta} \cdot G = [\delta_\xi, \frac{\partial G}{\partial \bar{\eta}}] \cdot G = [\delta_\xi, -\bar{K}\eta] \cdot G = \bar{\xi} \cdot G. \tag{86}$$

The local operator of coordinate gets the shift

$$\delta_\xi x_\mu \cdot G = (-i\xi\sigma_\mu\bar{\theta} + i\theta\sigma_\mu\bar{\xi}) \cdot G,$$

and we see that the considered BRST-transformation makes the group of supersymmetry in the superspace $\mathfrak{R}^{4|4} = \{x_\mu, \theta, \bar{\theta}\}$.

Indeed, the action of commutator on the partition function has the form

$$[\xi Q, \bar{Q}\bar{\xi}] \cdot G = 2\xi\sigma^\mu p_\mu\bar{\xi},$$

where we have introduced the notation

$$\delta_\xi = \xi Q + \bar{Q}\bar{\xi}.$$

Taking into account the anti-commutative properties of transformation parameters ξ, $\bar{\xi}$, we get

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} p_\mu.$$

A construction of Lagrangians invariant under the supertransformations in the theory of interacting fields can be performed in the ordinary technique of superfields, wherein the $\theta\theta \cdot \bar{\theta}\bar{\theta}$-components are transformed as full divergences and, hence, produce the invariant Lagrangians.

As for the physical meaning of such the derivation of supertransformations, we deal with the fact that the choice of reference zero-point at the trajectory of free spinor parametrized by τ is conventional, and it does not carry any measurable information (see Fig.1).

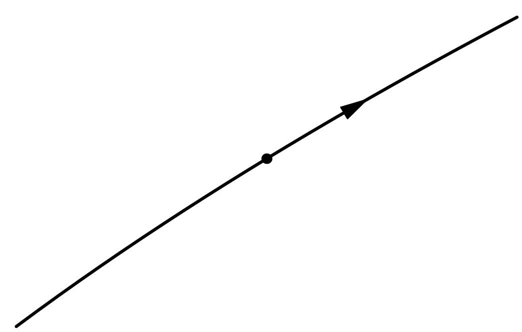

Figure 1: The displacement of zero-point at the trajectory of free spinor.

It is rather evident that under the displacement of reference point along the trajectory the physical observables do not change, i.e. there is an invariance, while the spinor coordinates and its wave function θ acquire the variations

$$\begin{aligned} \delta_\tau x_\mu &= v_\mu \, d\tau, \\ \delta_\tau \theta &= -\mathrm{i}\, d\tau\, \theta. \end{aligned} \tag{87}$$

The transformations of supersymmetry solve the inverse problem: at the fixed variation of spinor wave-function

$$\delta_\xi \theta = \xi = \delta_\tau \theta$$

in accordance with (87) one can find the change of spinor coordinates depending on the infinitesimal value ξ, which implicitly gives the shift of trajectory parameter τ. We have shown that such the solution can be constructed in the explicit form implying the introduction of supersymmetric transformations in the superspace if the spinor is global.

Thus, the supersymmetry has to be considered as necessary ingredient in the theory of interactions. Items on the structure of supersymmetric multiplets and spontaneous breaking of supersymmetry need a model study.

6 Discussion

The standard model of particle interactions, which finds an increasing number of its precise experimental confirmations, is built on the principle of local gauge group acting on the spinors, so that the Lagrangian remains invariant under the introduction of gauge vector fields. This theory is stable under the quantum loop corrections, since they do not lead to a necessary introduction of infinite number of new physical quantities, and the renormalizability provides the description of quantum fields in terms of their masses and charges, originally involved in the theory in the formulation at the classical level. The Einstein–Hilbert theory of gravitation[19] stands in a separate point, since it is based on no gauge principle with the vector fields mediating the interaction, but it operates with the metric tensor fields of spin 2, and in the calculations of quantum loops the theory of gravity generally needs to introduce an infinite number of new physical quantities, since it is not renormalizable. Moreover, the intermediate vector fields lead to a repulsive force for charges with the same sign and to a attraction for charges of different sign, so that trivial attempts to derive the Lagrangian of gravity on the basis of gauge principle with a group not connected with the space-time characteristics of particles, are certainly disfavored because the gravity does not distinguish the charges of sources by their signs. The nonrenormalizability of Einstein gravity is, first, a consequence of form for the interaction of space-time metrics with the energy-momentum tensor. Since the dimension of energy-momentum tensor is equal to four in the energy units, the rescaling of metrics to the canonical unit dimension of bosonic field[20] results in that the coupling constant should be equal to an inverse mass, so that loop corrections get increasing powers of divergency. Second, after the rescaling to the dimensional bosonic field of metrics, the factor providing the invariance of space-time volume measure, $\sqrt{-g}$, has a dimension equal to 2 and involves an appropriate multiplying factor as a coupling constant of dimension minus 2, that leads to the increase of divergency powers in the loop calculations with the matter, too. So, the one loop corrections require the introduction of curvature tensor squared, that implies the presence of higher space-time derivatives for the metrics as well as new coupling constants and cosmological term [11]. Moreover, the Einstein theory of gravity has intrinsically got an original scale of energy, that is determined by the dimensional gravitational constant, while in the gauge theories the presence of intrinsic scale takes place only in the spontaneous breaking of vacuum symmetry, the condensation of gauge fields and the renormalization group breaking of conformal invariance. On the other hand, the fundamental scale as in the gravitation can point to that there is a dimensional quantity in the primary theory that could be naturally involved in theories of nonlocal extended objects, for example, in the string theory. In this respect, the quantum theory of gravity can be constructed with no connection with the gauge principle and renormalizability. Some implications of such programme are well represented in modern studies of M-theory [12]. We emphasize also that the gauge interaction in extended dimensions leads to a necessary introduction of fundamental scale of energy, since the coupling constant becomes dimensional, that seems to be not attractive from the logical point of view because, for example, the consideration of scalar complex field, i.e. simple operation with the complex numbers, involves a necessary fundamental length. So, a parting with the powerful

[19] See a modern textbook by W.Siegel [7].

[20] In the momentum space of k, the propagator of such field behaves as $1/k^2$ with no other dimensional parameters.

features of gauge theories in four dimensions toward the theories in extended dimensions does not seem to be crucial. In another respect, a conversion to the theories of extended objects makes not only a logical step forward a denying the completeness of local quantum field theory, but also a more wide usage of mathematical language with all of its beauty and complication of analysis. In the present paper we have attempted to keep the quantum gravity in the framework of local quantum field theory by exploring a gauge symmetry possessing some nontrivial space-time properties and the gauge field condensation implying the spontaneous breaking of gauge symmtery.

We have studied the equivalence of spinor algebra representations as the gauge symmetry and found that the corresponding current by Noether for the Weyl spinors has occurred the current of interaction with the spin-connection. The global invariance of Lagrangian has been provided by the introduction of compensating gauge transformation for the auxiliary field of tetrad. The local gauge group has led to the introduction of nonabelian gauge fields, for which the invariant Lagrangian has been defined. Then we have considered the example of nonzero vacuum fields, which have caused the spontaneous breaking of gauge symmtery due to the condensation and given the effective low-energy Einstein–Hilbert action of gravity with the massless modes of spin 2, while the other degrees of freedom for the gauge field have got the masses. We have shown that the most aesthetic variant of modelling the vacuum structure has involved the fields determining the dual strength tensor of gauge field, so that in this way, the mechanism for the cancellation of cosmological constant has had the most natural form due to the symmetry between the gauge vacuum fields and those of dual strength tensor. In the other mechanism, the cancellation of cosmological term has taken place due to the condensate of spinor field, that has to be tuned with the condensates of gauge fields. This fact could suggest the underlying supersymmetry between the fermionic and bosonic fields. We have shown that the necessary introduction of supersymmetry has followed from the BRST-generalization of evolution operator for the Weyl spinor, since the physical observables of free Weyl particle have not to depend on the choice of time reference-point on the trajectory. Further we have shown which changes have been brought into the theory by investigating the multiplet of Weyl spinors composing the Dirac bispinor. In this case, the additional internal gauge symmetry including the standard model electroweak symmetry has occurred.

The most challenging problems of offered approach are a consistent construction of perturbation theory, a canonical quantization, a prove of renormalizability, a derivation of asymptotic freedom[21], a study of quantum anomalies and a consequent motivated analysis of vacuum structure.

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[21] We can easily find the expression for the structure constants of group under study from the definition of covariant derivative in the adjoint representation (33). So, for f^K_{PM}, where the anti-symmetric group indices are equal to $K = kl$, $P = pq$, $M = mn$, we get

$$f^{kl}_{pq,mn} = \frac{1}{2}(\delta^k_p\delta^l_n\eta_{qm} - \delta^l_p\delta^k_n\eta_{qm} - \delta^k_q\delta^l_n\eta_{pm} + \delta^l_q\delta^k_n\eta_{pm} - \delta^k_p\delta^l_m\eta_{qn} + \delta^l_p\delta^k_m\eta_{qn} + \delta^k_q\delta^l_m\eta_{pn} - \delta^l_q\delta^k_m\eta_{pn}).$$

Then we can calculate the Casimir operator C_A, defined by $C_A\, I_{PB} = f^K_{PM} f^K_{BM}$, where $C_A = 8$, while the group unity in the adjoint representation is given by $I_{PB} = \frac{1}{2} S_{pq,bc}$ in terms of S defined in (32). Therefore, we can expect that up to the one loop accuracy of pure gauge theory, the β function is negative, and it is equal to $\beta(g^2_{\rm Pl}) = \frac{dg^2_{\rm Pl}}{d\ln\mu^2} = -\frac{11}{3}\, C_A \frac{g^4_{\rm Pl}}{4\pi}$, pointing to the asymptotic freedom.

References

[1] J.Wess, J.Bagger, “Supersymmetry and supergravity” – Princeton: Princeton University Press, 1983.

[2] K.S.Stelle, Phys. Rev. **D16**, 953 (1977).

[3] R.Utiyama, Phys. Rev. **101**, 1597 (1956).

[4] E. Gabrielli, Annals Phys. **287**, 229 (2001) [arXiv:hep-th/9909117], Phys. Lett. B **258**, 151 (1991).

[5] A.A.Slavnov, L.D.Faddeev, “Introduction to quantum theory of gauge fields” – Moscow: Nauka, 1978 [in Russian].

[6] A.Palatini, Red. Circ. Mat. Palermo **43**, 203 (1919).

[7] W.Siegel, preprint YITP-99-67, hep-th/9912205 (1999).

[8] E.Schrödinger, “Space-time structure” – Cambridge: Cambridge University Press, 1950.

[9] L.D.Faddeev, Uspekhi Fiz. Nauk **136**, 435 (1982).

[10] A.D.Linde, “Physics of elementary particles and inflation cosmology” – Moscow: Nauka, 1990 [in Russian].

[11] N.D.Birrell, P.C.W.Devies, “Quantum fields in curved space” – Cambridge: Cambridge University Press, 1982.

[12] J.H.Schwarz, N.Seiberg, Rev. Mod. Phys. **71**, S112 (1999);
M.B.Green, J.H.Schwarz, E.Witten, “Superstring theory” – Cambridge: Cambridge University Press, 1987.